region $2\pi l/\lambda = 180°$ must be avoided in either path length. Thus, if the phase shifter were to be used in a phased array of much length and large sweep angle, many 90° bits would have to be used for the longer time-delay paths. When the switched-line phase shifter is used with Schiffman constant phase-delay lines, the long lengths of the lines constrain the constant phase-shift bandwidth to about $\frac{1}{2}$ octave. The new lumped-element high-pass low-pass phase shifter gives a very good constant phase shift for $\Delta\phi \leq 90°$. A practical octave bandwidth constant phase-shift phase shifter would use the new type phase shifters for all but the 180° bit, which could be a reflection device made using a very carefully matched quarter-wavelength 3-dB coupler and a pair of diodes.

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Equivalent Network for Interacting Thick Inductive Irises

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Abstract—**An equivalent network is presented for symmetric inductive irises in rectangular waveguides. This model exactly describes the effects of finite thickness and interaction via higher order modes due to the presence of neighboring irises, as in practical waveguide filters.**

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Introduction

THE NECESSITY of finding an exact equivalent network representation for inductive irises having finite thickness and possibly interacting via higher order modes arises in the design of high-precision waveguide filters. Current design practice assumes the irises as infinitely thin and noninteracting. A thickness cor-

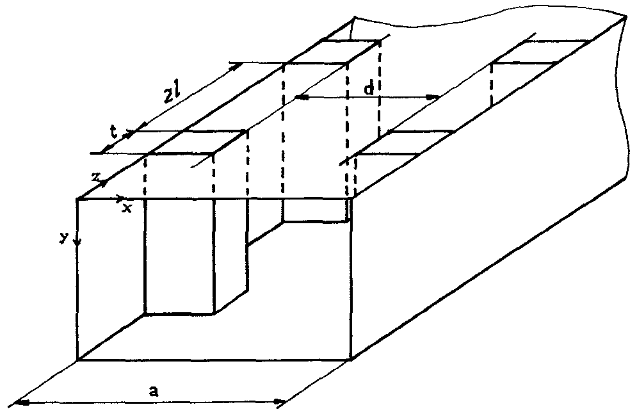


Fig. 1. Geometry (top view).

rection is then introduced which is strictly valid in the case of very thick irises only.

Account of the effect of higher order mode interaction is normally not taken. This results in incorrect values of intracavity coupling and "effective" cavity length. The resulting deviations are then compensated by means of tuning elements such as posts or screws.

This paper introduces an accurate equivalent network that considers the combined effect of finite thickness and interaction from the start. Elegant solutions for the limiting vases of the general problem of the thick interacting inductive iris are well known.

The variational approach to the problem of one infinitely thin inductive iris is discussed by Collin [1]. A fortunate choice of the trial field over the iris aperture, originally due to Schwinger [2], allows one to express the equivalent susceptance as a fast converging series from which the "quasi-static" limit is directly recovered. The general case of an arbitrary incident field has been treated by Palais [3] using the same type of trial field functions. One (isolated) thick iris has also been studied by Collin [1]. He assumes this time, an eigenmode expansion of the iris, as the trial field in his variational solution. The above-mentioned contributors use an "aperture" formulation of the discontinuity problem.

The interaction of two infinitely thin inductive irises has been discussed by Palais [4] employing an "obstacle" formulation. In this paper the aperture formulation will be adopted and the Schwinger functions will be assumed for the trial field. This allows the solution of the infinitely thin iris to be used directly in conjunction with the equivalent network representation of the thick iris. The interaction effect is introduced by a suitable modification of the modal admittances appearing in the expression for the susceptance of the infinitely thin iris.

The situation under study is depicted in Fig. 1 for a two-element filter. The irises are taken to be symmetrical since this is a case of practical importance and the asymmetric case involves essentially no new features.

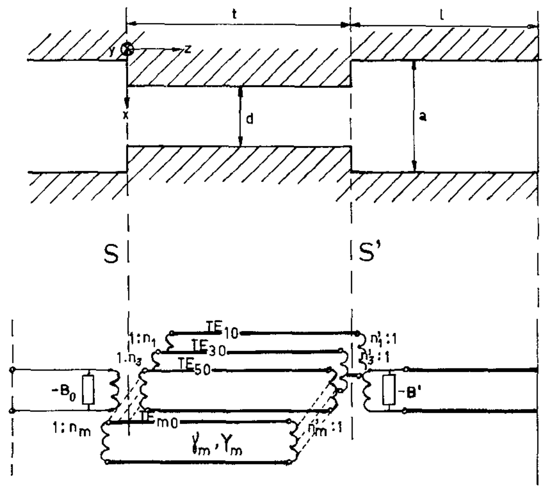


Fig. 2. Equivalent network representation.

Making use of the mirror symmetry with respect to the plane $z=t+l$ we shall split the problem into an even and an odd part by locating a magnetic and an electric wall, respectively, at the plane of symmetry.

Let us assume that a TE_{10} mode with unit amplitude is impinging from the left. All other modes in the main guide and all modes in the iris aperture are below cutoff. Because of the symmetry in the y-direction, only TE_{n0} modes with odd n's are excited. An intuitive and direct way of obtaining a network representation is shown in Fig. 2. All occurring modes in the thick iris are carried by separate transmission lines coupled by transformers at the interfaces S and S'. The fields over S and S' in Fig. 2 are assumed to be two expansions of orthogonal functions over the aperture with undetermined coefficients. In Fig. 2, B_0 represents one half of the susceptance of an infinitely thin iris in the absence of interaction, as computed from the field assumed over S. B' is the equivalent of B_0 in the presence of the magnetic/electric wall at $z=t+l$. The transformer ratio associated with a certain iris eigenmode is given by the ratio of the "scalar product" (in the functional sense) of the assumed field with the relevant modal field and the product of the assumed field with the exciting field. Since the guide is infinite in the negative z-direction, but is terminated by an electric or magnetic wall at $z=t+l$, the transformer ratios on the two sides are unequal. The computation of B' is similar to that of B_0 but account is taken of the electric or magnetic wall termination in the modal admittances. The whole equivalent network is determined by minimizing the power stored in the even- and odd-mode one-port networks of Fig. 2 for a unit excitation amplitude. The case of single or noninteracting thick irises is also treated. The solution in this case can be conveniently written in the form of a perturbation from the quasi-static value. This provides a rather compact alternative solution to the one already available [3].

Numerical and experimental results are compared for one single thick iris and two identical irises in the presence of interaction.

Theory: Two Interacting Irises

In the network representation of Fig. 2, $B_0(Y=-jB_0)$ is one half of the susceptance of an infinitely thin symmetric iris normalized to the characteristic admittance of the TE_{10} mode in the feeding guide [1].

$$B_0 = \frac{\pi}{a\beta}\int_0^\pi \int_0^\pi F(\theta)\cdot F(\theta')\cdot\Bigg[\sum_{n=1}^{\infty}\frac{1}{n}\cdot\cos(n\theta)\cdot\cos(n\theta') - \alpha^2\cos(\theta)\cdot\cos(\theta') + \sum_{n=3}^{\infty}\frac{1}{n^2}\cdot\left(\frac{a}{\pi}\Gamma_n - n\right)\cdot\sum_{r,s=1,3}^{n} P_{nr}P_{ns}\cos(r\theta)\cdot\cos(s\theta')\Bigg]d\theta\cdot d\theta' / \cdot\left[\alpha\int_0^\pi \cos(\theta)\cdot F(\theta)d\theta\right]^2 \tag{1}$$

where the variable θ is defined by

$$\cos\frac{\pi x}{a} = \alpha\cos\theta. \tag{2}$$

Further

- $E(x)$ the trial field on the aperture;
- $F(\theta)$ $=dE/dx$;
- β the propagation constant of the TE_{10} mode in the guide;
- Γ_n the wave number of the TE_{n0} mode (with n odd);
- α $=\sin(\pi d)/(2a)$;

and the coefficients P_{nr} of the expansion

$$\cos\left(\frac{n\pi x}{a}\right) = \sum_{r=1}^{n} P_{nr}\cos(r\theta) \tag{3}$$

are given in [1], [3].

By taking the trial function $F(\theta)$ as

$$F(\theta) = \sum_{k=1}^{2N+1}\lambda_k\cos(k\theta) = \sum_{k=1}^{2N+1}\lambda_k T_k\left(\left(\cos\left(\frac{\pi x}{a}\right)\right)\Big/\alpha\right) \tag{4}$$

with $\lambda_1=1$ and T_K being the Chebyshev polynomial of the first kind, we see that (1) reduces to

$$B_0(\lambda_1\cdots\lambda_{2N+1}) = \frac{\pi}{a\beta}\cdot\frac{1}{\alpha^2}\Bigg[1-\alpha^2+\sum_{n=3,5,\cdots}^{\infty}\frac{1}{n^2}\cdot\left(\frac{a}{\pi}\Gamma_n - n\right)\cdot\sum_{i,j=1,3,\cdots}^{2N+1}\lambda_i\lambda_j P_{ni}P_{nj} + \sum_{i=3,5,\cdots}^{2N+1}\frac{\lambda_i^2}{i}\Bigg]. \tag{5}$$

Neglecting the sum, in B_0 yields the quasi-static approximation

$$B_0 = \frac{\pi}{a\beta}\cot^2\left(\frac{\pi d}{2a}\right). \tag{6}$$

The transformer ratios on the left-hand side of the iris (Fig. 2) are given by

$$n_m = \frac{V_m}{V_0}, \qquad (m = 1, 3, \cdots) \tag{7}$$

where V_m is the voltage coupling of the mth-aperture eigenmode $\sqrt{2/d}\,\psi_m(x)$ with the trial field and V_0 is the voltage coupling of the trial field with the impinging TE_{10} wave. Thus

$$n_m = \sqrt{a/d}\cdot\frac{\int_{(a-d)/2}^{(a+d)/2} E(x)\cdot\Psi_m(x)\cdot dx}{\int_{(a-d)/2}^{(a+d)/2} E(x)\cdot\sin\left(\frac{\pi x}{a}\right)\cdot dx} = \frac{2}{\alpha\sqrt{ad}}\cdot\left(\sum_{k=1}^{2N+1}\lambda_k a_{mk}\right) \tag{8}$$

with

$$a_{mk} = \frac{d}{\pi m}\int_0^\pi \cos k\theta\cos\frac{m\pi}{d}\left(x(\theta) - \frac{a-d}{2}\right)d\theta. \tag{9}$$

On S' we assume the trial field $E'(x)$ such that

$$\frac{dE'(x)}{dx} = F'(\theta) = \sum_{k=1,3,\cdots}^{2N+1}\mu_k\cdot\cos(k\theta) \tag{10}$$

with $\mu_1=1$.

The transformers n_m' on the right-hand side of the iris are given by (8) with the λ's replaced by the μ's. The thick iris is represented by the parallel connection of an infinite number of transmission lines of length t (below cutoff) between ideal transformers.

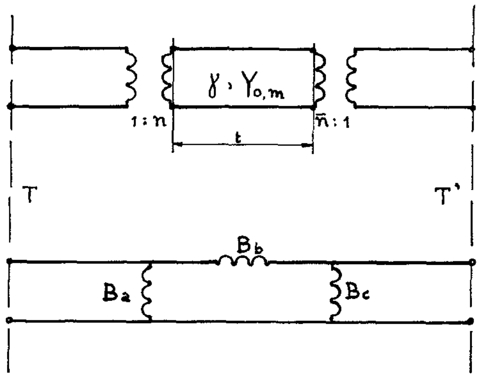
The equivalent pi-network representation for a length t of transmission line (below cutoff) with propagation constant γ is

$$y_{11} - y_{12} = -jB_a = \frac{\gamma}{\beta}(\coth(\gamma t) - \operatorname{csch}(\gamma t)) \tag{11a}$$

$$y_{12} = -jB_b = \frac{\gamma}{\beta}\operatorname{csch}(\gamma t). \tag{11b}$$

This equivalent network and the elimination of the transformers are shown in Fig. 3. The thick iris is represented by the parallel connection of the equivalent circuit of each mode and the overall equivalent pi network has the following elements:

$$y_{11} - y_{12} = -jb_a = \sum_{m=1,3,5,\cdots}^{\infty}\frac{\gamma_m}{\beta}\cdot\left[n_m^2\coth(\gamma_m t) - n_m' n_m\operatorname{csch}(\gamma_m t)\right] \tag{12a}$$



$$-jB_a = \frac{\gamma}{\beta}\left[n^2 \coth(\gamma t) - nn' \operatorname{csch}(\gamma t)\right]$$

$$-jB_b = \frac{\gamma}{\beta} nn' \operatorname{csch}(\gamma t)$$

$$-jB_c = \frac{\gamma}{\beta}\left[n' \coth(\gamma t) - nn' \operatorname{csch}(\gamma t)\right]$$

Fig. 3. Equivalent network for one iris mode.

$$y_{12} = -jb_b = \sum_{m=1,3,5,\cdots}^{\infty} \frac{\gamma_m}{\beta} \cdot n_m n_m' \operatorname{csch}(\gamma_m t) \quad (12b)$$

$$y_{22} - y_{12} = -jb_c = \sum_{m=1,3,5,\cdots}^{\infty} \frac{\gamma_m}{\beta} \cdot \left[n_m^{2'} \coth(\gamma_m t) - n_m n_m' \operatorname{csch}(\gamma_m t)\right]. \quad (12c)$$

The occurrence of infinite m sums in (12) originates from the fact that we have expressed the trial fields not as combinations of actual aperture eigenmodes $\psi_m(x)$, but rather as expansions of the type (2).

Although these two sets of functions do not coincide, their similarity is apparent. Already for relatively small values of $|m$ and $k|$, $a_{mk}^2 \approx 0$, and thus the series in (12) converges quite rapidly. The computation of B' from the trial field on S' proceeds in a manner similar to that of B_0. In this case, however, account must be taken of the fact that now the transmission lines carrying the modes are not terminated by their characteristic admittances, but rather by a magnetic or an electric wall located at $z=t+l$.

The expression for B' is then obtained by replacing the λ's in (5) with the μ's and each Γ_n with

$$\Gamma_n \cdot \begin{Bmatrix} \tanh(\Gamma_n l) \\ \coth(\Gamma_n l) \end{Bmatrix}$$

in the even/odd case, respectively.

A length of transmission line with unit characteristic admittance terminated by an open/short circuit is the equivalent network for the ground mode between $z=t$ and $z=t+l$.

Its contribution to the admittance is

$$y = jb = \begin{cases} j \tan(\beta l) \\ -j \cot(\beta l). \end{cases} \quad (13)$$

The total driving-point susceptance for the even/odd case is therefore

$$B = B_0 + b_a + b_b \cdot (b_c + B' - b)/(b_b + b_c + B' - b). \quad (14)$$

The driving-point admittance for the even/odd one-port junction has been written in terms of the yet undetermined $N+2$ amplitudes $\lambda_3, \lambda_5, \lambda_{2N+1}; \mu_3 \cdots \mu_{2N+1}$. Its actual value is obtained by minimizing the power stored in the junction for a unit voltage excitation, i.e., minimizing the function $B(\lambda_3, \lambda_5 \cdots \lambda_{2N+1}; \mu_3, \mu_5 \cdots \mu_{2N+1})$ or equivalently solving the simultaneous equations

$$\frac{\partial B}{\partial \lambda_i} = \frac{\partial B}{\partial \mu_i} = 0, \qquad i = 3, 5, \cdots, 2N+1 \quad (15)$$

for the even/odd problem by means of standard numerical methods.

One Isolated Thick Iris

The susceptance for one isolated thick iris is given by:

$$\begin{aligned} B(\lambda_3, \lambda_5, \cdots, \lambda_{2N+1}) &= B_0 + \sum_{m=1,3,\cdots}^{\infty} \frac{\gamma_m}{\beta} \cdot \begin{Bmatrix} \tanh((\gamma_m t)/2) \\ \coth((\gamma_m t)/2) \end{Bmatrix} \\ &= \frac{\pi}{a\beta} \cdot \frac{1-\alpha^2}{\alpha^2} - \sum_{i,j=1,3,\cdots}^{2N+1} \lambda_i \lambda_j \cdot \left[\frac{\pi}{a\beta} \cdot \frac{1}{\alpha^2}\right. \\ &\quad \cdot \left[\sum_{n=3,5,\cdots}^{\infty} \frac{1}{n^2} \cdot \left(n - \frac{a\Gamma_n}{\pi}\right) \cdot P_{ni} P_{nj} + \frac{1}{i} \delta_{ij}\right] \\ &\quad \left. - \frac{4}{ad\alpha^2} \sum_{m=1,3,\cdots}^{\infty} a_{mi} a_{mj} \cdot \frac{\gamma_m}{\beta} \begin{Bmatrix} \tanh(\gamma_m t/2) \\ \coth(\gamma_m t/2) \end{Bmatrix}\right] \\ &= \frac{\pi}{a\beta} \cot^2\left(\frac{\pi d}{2a}\right) - \Delta B_0 + \Delta B_t. \end{aligned} \quad (16)$$

The first term above represents the quasi-static contribution to the thin-iris susceptance, the second term ΔB_0 is the contribution of higher order modes to the thin-iris susceptance, and the third term ΔB_t is due to the thickness effect. Also we recognize that $\Delta B_{0,t}$ in (16) are quadratic forms in λ

$$\Delta B_{0,t} = \lambda^t M_{0,t} \lambda \quad (17)$$

where

$$\lambda^t = (\lambda_1, \lambda_3, \cdots, \lambda_{2N+1}), \qquad \lambda_1 = 1 \quad (18)$$

and the matrices M_0, M_t are real symmetric and positive definite, and can thus be diagonalized simultaneously. This allows us to express our solution in a very convenient form. In fact, as a consequence of Courant–Fisher's theorem [4], the maximum value of $\Delta B = \Delta B_0 - \Delta B_t$ is given by the maximum eigenvalue σ of $M = M_0 - M_t$.

The total even/odd susceptance is then

$$B = B_{\text{q.static}} - \sigma. \quad (19)$$

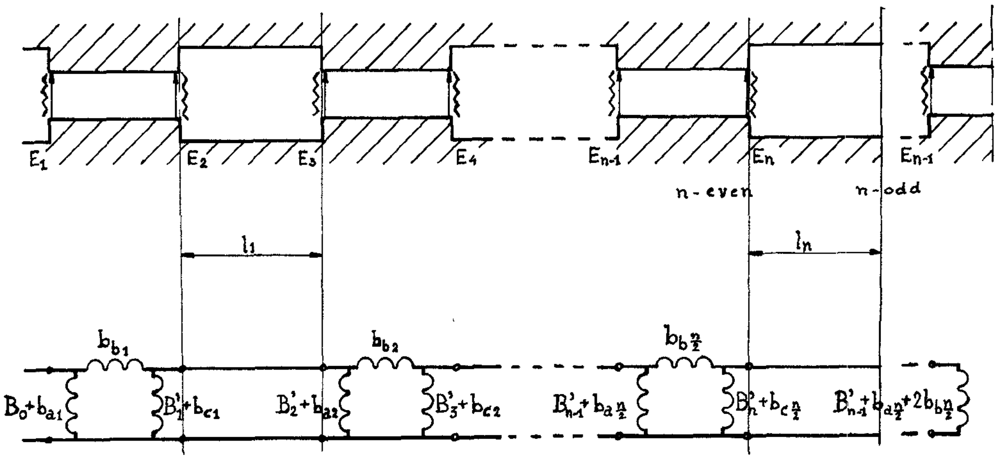


Fig. 4. Multiple iris configuration.

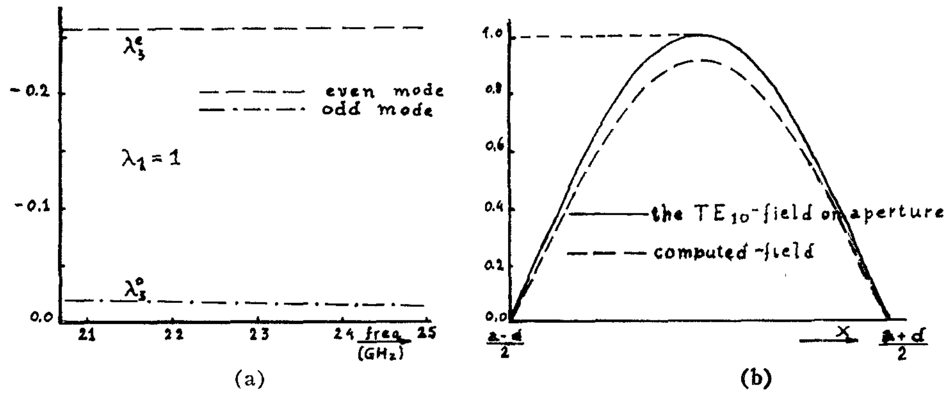


Fig. 5. (a) Normalized modal amplitudes in single thick iris ($\lambda_1 = 1$). (b) E field configuration on aperture.

Also, the amplitude vector λ is the eigenvector of M relative to σ. The determination of σ and λ is a very fast standard numerical procedure.

More than Two Irises in Cascade

The extension to symmetrical filters having more than two interacting irises with different apertures proceeds in general as illustrated in Fig. 4. Use is again made of the halfway symmetry of the filter to reduce the problem to that of an even- and odd-case one-port junction. A trial field is assumed at every interface. Since a two-term trial field gives quite accurate results, the number of variables involved is not excessive up to a moderately large number of irises. Looking from one iris towards the following one, the aperture of the latter can be considered closed by a metallic screen super-imposed by the appropriate magnetic current distribution (e.g., E field). The susceptance located at the first aperture is thus B' (for the odd case, now being the length of the waveguide between the two irises).

If the filter has an odd number of irises, the halfway symmetry plane is located on the midplane of a thick iris and the terminal susceptance is given by (16). The overall susceptance (14) is obtained by standard network analysis. For n irises and M terms in the trial field, the problem is eventually reduced to the solution of a system of $2\times(M-1)\times(n/2$ for n even) $(n+1/2$ for n odd) simultaneous equation for the even and the odd mode separately. Because of linearity, the actual field $E(x)$ over each aperture is given by

$$E(x) = \tfrac{1}{2}(E_{\text{even}}(x) + E_{\text{odd}}(x)). \tag{20}$$

Numerical and Experimental Results

The reflection coefficient for a single thick iris having the following dimensions, in centimeters,

guidewidth	$a = 1.0668$,
iris aperture	$b = 0.447$,
iris thickness	$t = 0.050$,

was computed over the band 18–25 GHz (K band) corresponding to $2.50 \geq \lambda_g/a \geq 1.36$.

A two-term trial field

$$F(\theta) = \cos(\theta) + \lambda_3 \cos(3\theta) \tag{21}$$

was employed. The relative amplitudes of $\lambda_3{}^e$ and $\lambda_3{}^o$ (even and odd, respectively) are given in Fig. 5(a) over the whole frequency range. Fig. 5(b) compares the computed configuration of $E(x) = \int F(\theta(x))\, dx$ with that of the TE_{10} aperture eigenmode at $f = 22.981$ GHz. The

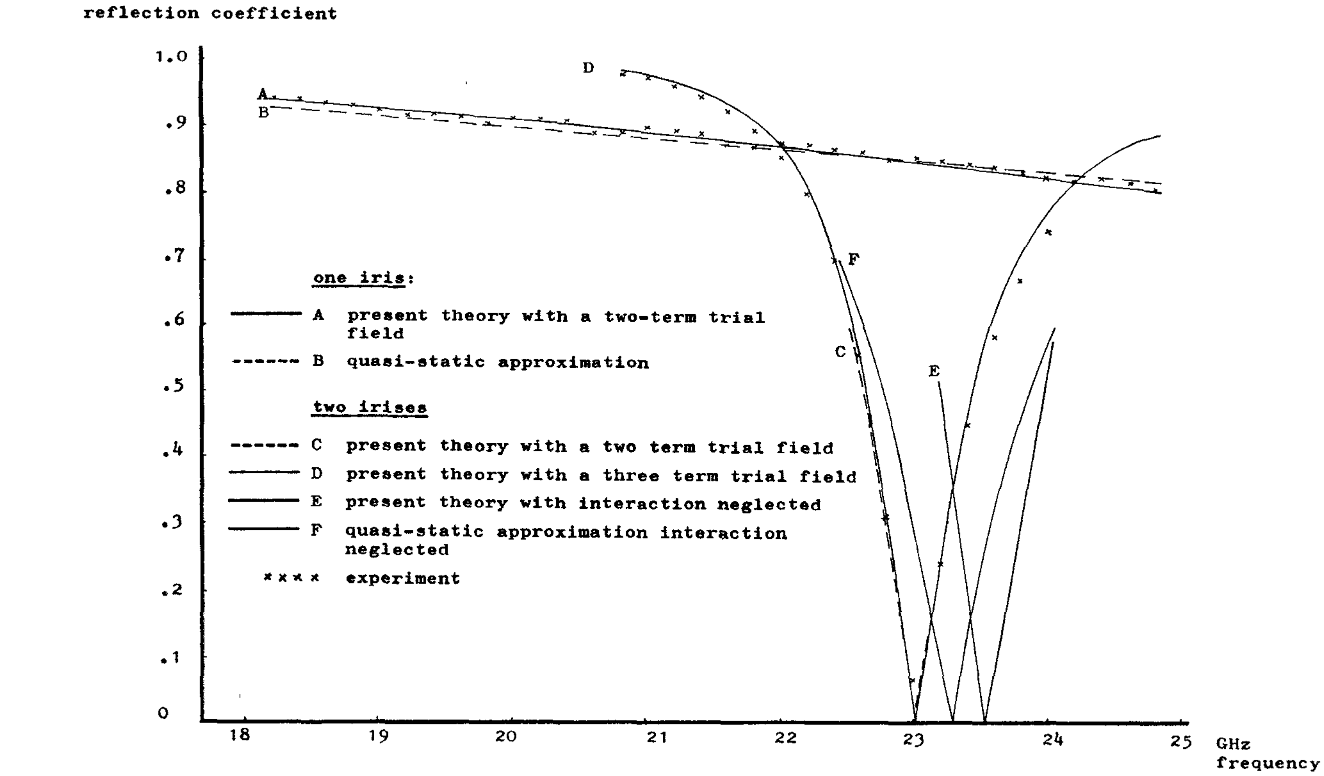


Fig. 6. Computed and experimental reflection modulus for one and two irises.

computed values of the modulus of the reflection coefficient of the iris are compared in Fig. 6 (curve A) with the experimental ones obtained by slotted-line measurements.

Curve B in the same figure was obtained by taking the quasi-static approximation for the iris susceptance and accounting for thickness as described in [5]. The latter approach consists in considering the thickness effect due only to the fundamental mode of the aperture. For not too thin irises it is tantamount to reducing the transmission coefficient through a thin iris by a factor $\exp\left(-\pi t/d\cdot\sqrt{1-(2d/\lambda)^2}\right)$. It is seen that the modulus of the susceptance for an isolated iris is approximated fairly well by this method, although its frequency dependence is slightly incorrect.

In order to experimentally check the model in presence of interaction, a simple filter was built consisting of two identical irises with the dimensions given above spaced 0.651 cm apart. The modulus of the reflection coefficient was measured over the K band. These values, corrected for the losses for large values of the reflection coefficient, are compared in Fig. 6 with the numerical values obtained by means of a two-term trial field (curve C) and a three-term trial field (curve D). Curve E in Fig. 6 has been computed by means of (16) which is exact for a single iris assuming the two irises to be noninteracting, i.e., that the higher order modes excited at the location of the two irises are not coupled. This assumption leads to an overestimate of the magnitude of the "effective" susceptance, while the bandwidth of the resonance is correct.

Finally, curve F has been computed using the quasi-static approximation for infinitely thin irises with exponential thickness correction, neglecting higher order mode interaction. This is the method commonly used in the practical realization of filters.

It is apparent from E and F that neglecting higher order mode interaction is the most serious approximation involved in the existing method.

The amplitudes of the fields on S and S' obtained with the two- and three-term approximations at resonance frequency ($f=22.981$ GHz) are compared in the following table.

		S		S'	
		λ_3	λ_5	μ_3	μ_5
Two terms	even	−0.2446		−0.1095	
	odd	−0.1099		−0.2397	
Three terms	even	−0.3170	−0.1019	−0.1838	−0.0415
	odd	−0.1821	−0.0420	−0.3133	−0.1017

($\lambda_1=\mu_1=1$)

Fig. 7 further illustrates the results of the three-term trial-field computation. In Fig. 7(a) and (b) the relative amplitudes versus frequency are plotted, while in Fig. 7(c) and (d) the fields at resonance are compared with the distribution of the TE_{10} mode on the aperture on S and S', respectively.

Fig. 8 illustrates the effect of mechanical tolerances upon the filter characteristics. The "computer experiments" were performed by using a two-term field expansion.

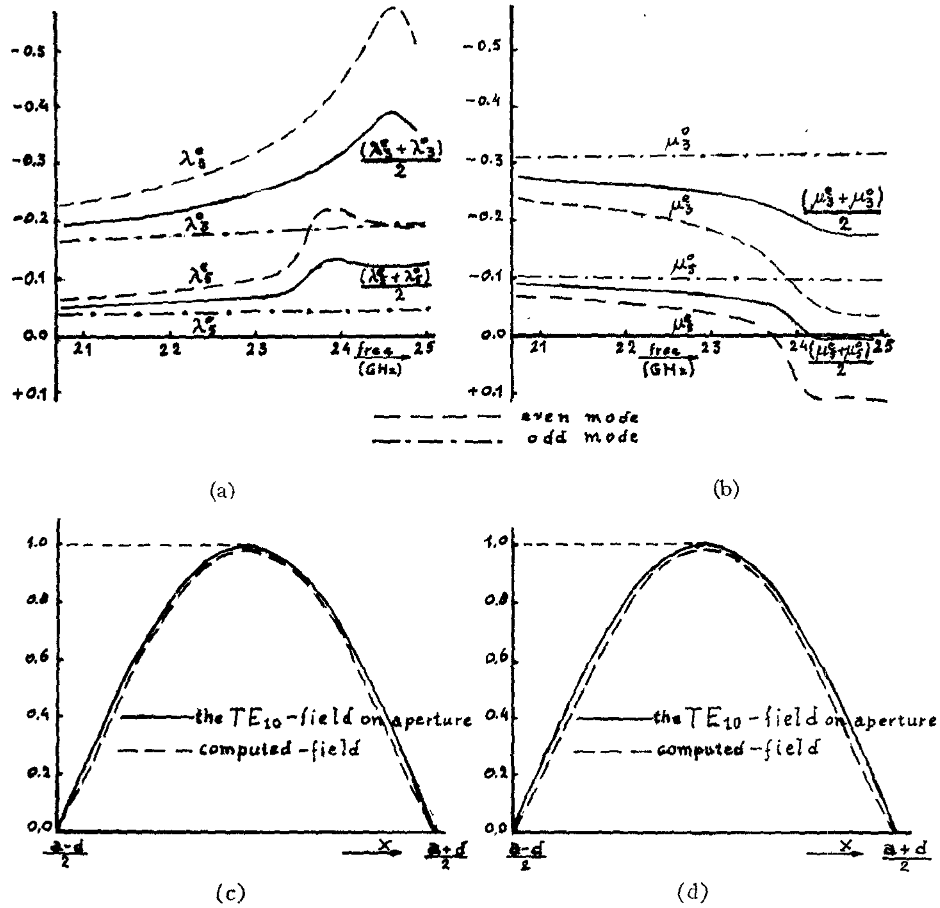
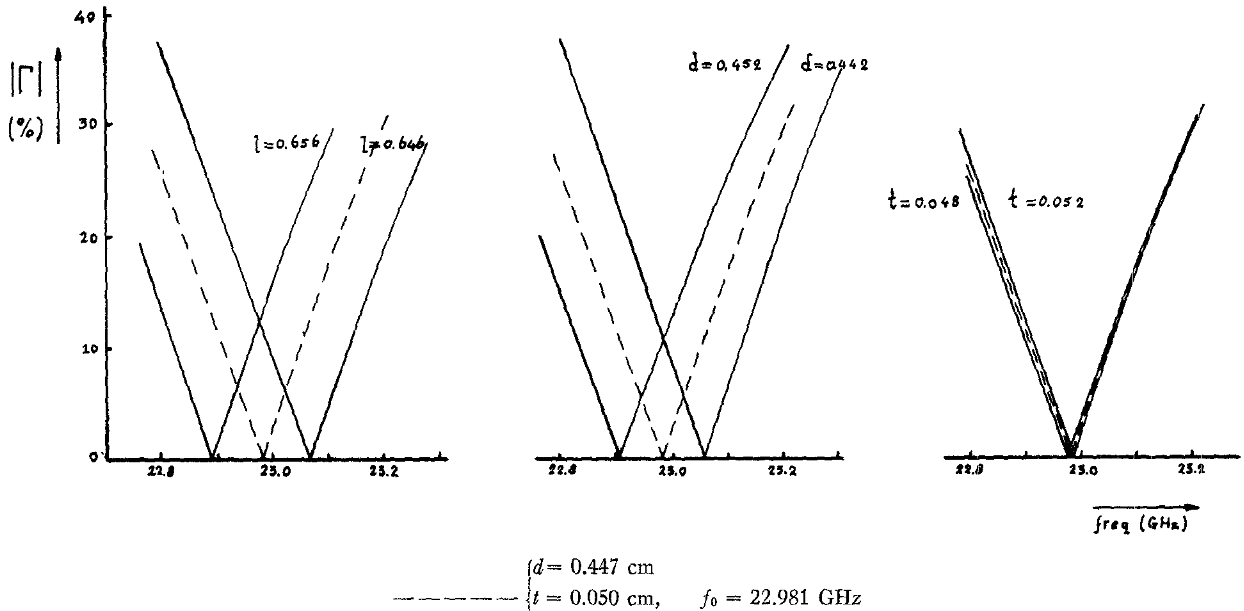


Fig. 7. (a) and (b) Normalized modal amplitudes for two thick interacting irises at resonance ($\lambda_1 = \mu_1 = 1$). (c) and (d) Field configurations at resonance.



------ $d = 0.447$ cm, $t = 0.050$ cm, $l = 0.651$ cm, $f_0 = 22.981$ GHz

Fig. 8. Effect of mechanical tolerances.

The following limiting values for the tolerances were assumed:

$$d = 0.447 \pm 0.005$$
$$t = 0.050 \pm 0.002$$
$$l = 0.651 \pm 0.005.$$

One parameter at a time was allowed to take its extremal values with the other two held constant. A number of similar computations, with different values for the iris aperture thickness and distance between irises, show that the first and the last parameters play a much more important role than thickness.

The above tolerances are arbitrary outer limits and, in fact, those allowed in the experimental models were much tighter. Also the frequency range was purposely chosen to be K band. At lower frequencies, the filter characteristics become increasingly less sensitive with the increasingly favorable ratio of mechanical tolerances to guided wavelength.

Conclusions

The simultaneous effect of finite thickness and interaction via higher order modes for symmetric inductive irises has been investigated by means of a variational approach.

An exact equivalent network is proposed whose elements can be derived by means of a simple computer procedure. The method is applicable to the analysis of waveguide filters having up to a moderate number of irises.

Computer results are in good agreement with measurements for two experimental models: A single thick iris, and two thick irises in the presence of higher order mode interaction.

Further, the theory illustrates the limits of the approximations involved in the current characterization of inductive irises.

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